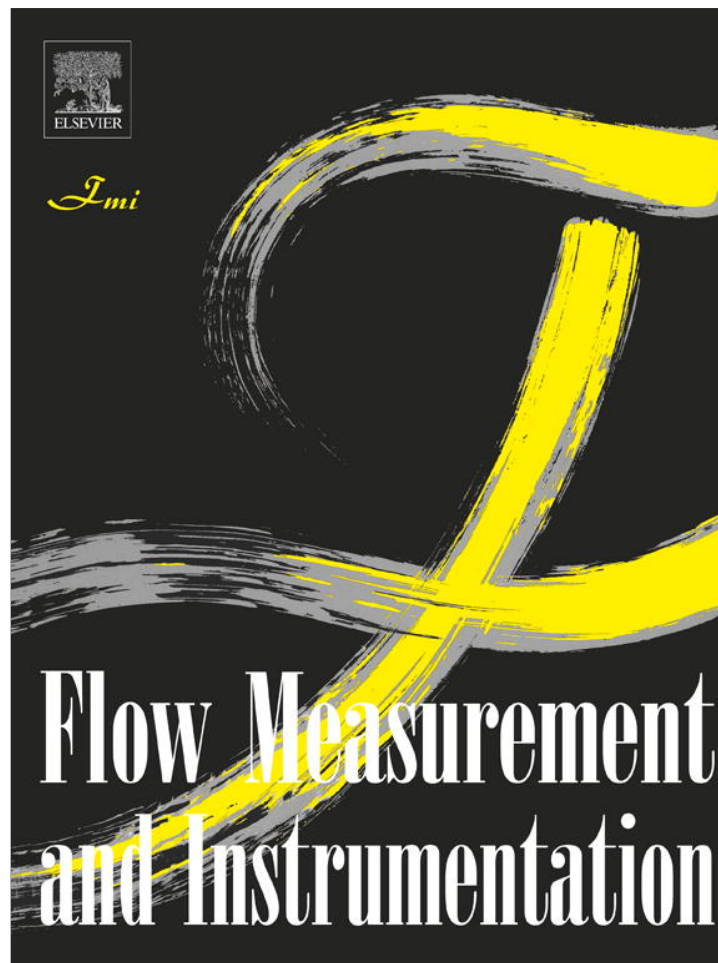


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## Study of flow at side weir in narrow flume using visualization techniques

Gorazd Novak<sup>a,\*</sup>, Daniel Kozelj<sup>a</sup>, Franc Steinman<sup>a</sup>, Tom Bajcar<sup>b</sup>

<sup>a</sup> University of Ljubljana, Faculty of Civil and Geodetic Engineering, Chair of Fluid Mechanics with Laboratory, Hajdrihova 28, Ljubljana, Slovenia

<sup>b</sup> University of Ljubljana, Faculty of Mechanical Engineering, Aškerčeva 6, Ljubljana, Slovenia

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### ABSTRACT

The purpose of the research was to quantify characteristics of a subcritical flow at a rectangular sharp-crested side weir in a rectangular main channel using non-invasive measuring techniques based on the visualization of the flow. Experiments were carried out in physical models, including nine different dimensions of the side weir and nine combinations of the inflow and tailwater level for each weir, amounting to 81 test runs. Velocity vector fields were measured in various horizontal planes along the side weir using a high speed digital camera and electrolysis-induced hydrogen bubbles as flow tracers. Recorded films were converted into sequences of images which were used for numerical calculation of local velocities. Components of velocity vectors were determined with great spatial and time resolution. Longitudinal profiles of water surface elevation at each side weir were determined using photos of laser-induced vertical section planes. Measured discharges and flow depths were used to formulate new equation for the side weir discharge coefficient using dimensional analysis. The principal results indicated that velocity distribution along the side weir was distinctly non-uniform, with various velocity ratios increasing along the crest. However, the calculated energy grade line was parallel to the main channel bed, indicating that only friction losses were present. The proposed equations for the side weir discharge coefficient gave results that were in good accordance with two other studies.

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### 1. Introduction

Side weirs are hydraulic structures for diverting discharge from a main channel to a lateral channel or a flood plain. They are widely used in irrigation, sewer, wastewater treatment and flood management systems. Flow over side weir is a case of spatially-varied flow with decreasing discharge and can be described with two alternative approaches, *i.e.* energy or momentum approach, which differ in assumptions concerning values of velocity distribution coefficients  $\alpha$  (*i.e.* kinetic energy correction coefficient), and  $\beta$  (momentum correction coefficient). Assuming constant specific energy across the weir and constant and uniform velocity distribution across the channel, Rosier [1] gives the general equation of the discharge  $q$  per unit length over the weir as:

$$q = 2/3C_d(2g)^{1/2}(h-p)^{3/2} \quad (1)$$

The performance of side weirs was investigated from the pioneering work of De Marchi [2], seminal work of Hager [3], to recent work of Emiroglu et al. [4]. Recent works based on the

constant energy approach are by Singh et al. [5], Swamee et al. [6], or Borghei et al. [7]. The effect of specific energy variation was considered by Yüksel [8] and Venutelli [9]. Representative works based on the momentum approach are by El-Khashab and Smith [10], Hager and Volkart [11], Lee and Holley [12] or May et al. [13].

The objectives of the present study were: (1) to quantify vector velocity fields using non-invasive visualization method and thus avoid flow perturbations caused by intrusive instrumentation, (2) to determine longitudinal water surface profile along the side weir using another non-contact visualization technique, and (3) to formulate a new equation for the discharge coefficient  $C_d$  using dimensional analysis, *i.e.* Buckingham  $\Pi$  theorem, which allows  $C_d$  to be expressed as a product of selected dimensionless ratios of various experimentally measured values. The present study is a continuation of research by Novak et al. [14]. Notations and a definition sketch of investigated side weirs are shown in Fig. 1:

In Fig. 1a  $L_z$  and  $z_z$  represent the sharp-crested rectangular weir for tailwater regulation. Fig. 1b shows velocities at cross section  $x$ .

### 2. Material and methods

The goals of this study were reached using mostly experimental setup described by Novak et al. [14] and a visualization

\* Corresponding author. Tel.: +386 14253460; mob.: +386 31549134; fax: +386 14269163.

E-mail addresses: [gorazd.novak@fgg.uni-lj.si](mailto:gorazd.novak@fgg.uni-lj.si) (G. Novak), [daniel.kozelj@fgg.uni-lj.si](mailto:daniel.kozelj@fgg.uni-lj.si) (D. Kozelj), [franci.steinman@fgg.uni-lj.si](mailto:franci.steinman@fgg.uni-lj.si) (F. Steinman), [tom.bajcar@fs.uni-lj.si](mailto:tom.bajcar@fs.uni-lj.si) (T. Bajcar).

**Nomenclature**

*Symbols*

$B$	width of the main channel [m]
$h$	flow depth in the centerline of the main channel [m]
$Fr$	Froude number
$L$	length of the side weir crest [m]
$p$	height of the side weir crest [m]
$Q$	discharge [ $\text{m}^3/\text{s}$ ]
$S_0$	main channel bed slope [-]
$u$	velocity component in $x$ direction [m/s]
$\bar{u}$	average velocity in $x$ direction [m/s]
$u_c^*(Q)$	uncertainty of flow measurement [%] (notation in accordance with ISO 1438:2008)
$u_c^*(\bar{u})$	uncertainty of velocity measurement [%](notation in accordance with ISO 1438:2008)

$v$	velocity component in transverse direction [m/s]
$x$	longitudinal direction, also distance from upstream end of the side weir [m]
$\alpha$	kinetic energy correction coefficient [-]
$\beta$	momentum correction coefficient [-]

*Indices*

1	upstream end of the side weir
2	downstream end of the side weir
s	side weir
w	wire (in connection with electrolysis-induced hydrogen bubbles)
z	weir for regulation of tailwater level

method introduced by Bajcar et al. [15]. In our previous study (Novak et al. [14]) results for velocity ratios  $u_s/\bar{u}$ , and coefficients  $\alpha$  and  $\beta$  from 18 test runs were presented, covering 3 combinations of inflow  $Q_1$  and tailwater weir  $z_z$  for each of the 6 side weirs (various crest length  $L$  and crest height  $p$ ) in the  $B=14$  cm wide channel. In the present study additional experimental data were used to determine velocity ratio  $v_s/u_s$ , energy grade line, and  $C_d$ . The range of experiments by Novak et al. [14] was extended to cover additional 63 tests, amounting to 81 tests, listed in Table 1. Ranges of main parameters are given in Table 2.

The experiments were carried out in two glass-walled 7.5 m long flumes with a minimal slope ( $S_0=0.05\%$ ), one 0.2 m wide, 0.5 m deep, and the other 0.5 m wide, 0.8 m deep. Plexiglass side weirs of different sizes were placed in these flumes with a rectangular main channel  $B=14$  cm and  $B=30$  cm wide, respectively. In the narrower channel 6 side weirs were studied, labeled  $L10\_p7.5$ ,  $L15\_p7.5$ ,  $L15\_p10$ ,  $L20\_p10$ ,  $L20\_p12$ , and  $L25\_p12$  with numbers representing dimensions in cm. Three more side weirs were examined in the wider channel:  $L50\_p12$ ,  $L75\_p14$ , and  $L100\_p20$  (again, dimensions in cm). Flow conditions were sub-critical and the lateral overflow  $Q_s$  was modular in all test runs. Adjustable rectangular sharp-crested weir was used to control the tailwater level and to provide sufficient overflow depth at side weir ( $h-p \geq 2$  cm) to avoid surface tension issues. In the narrower physical model, both inflow  $Q_1$  and flow  $Q_2$  remaining in the main channel downstream of the side weir were measured using thin-plate V-notch weirs. Side overflow  $Q_s$  was not measured, because it spilled directly into water tank below the flume,

and was calculated as  $Q_1-Q_2$ . In test runs with  $B=30$  cm the  $Q_s$  spilled into a 19 cm wide parallel lateral channel. For these tests, both  $Q_s$  and  $Q_2$  were measured using rectangular thin-plate weirs in accordance with ISO standard 1438:2008 [16] and its technical corrigendum [17], while the inflow  $Q_1$  was measured with a larger V-notch weir. The uncertainty of flow measurements  $u_c^*(Q)$  was calculated using the above standard and it amounted to  $u_c^*(Q)=2\%$ . Cross sections at  $x/L=0.5$  for both sets of experiments are given in Fig. 2.

In Table 1, notation of test runs in the narrower flume indicates a selected basic combination of  $Q_1$  and  $z_z$  (notation “var 0”), and whether the  $Q_1$  was varied at fixed position  $z_z$  (variants “Q–” to “Q+” with – or+ meaning lower or greater inflow), or position  $z_z$  was varied at fixed  $Q_1$  (variants “Z–” to “Z+” with – or+ meaning smaller or greater height  $z_z$ ). Notation “\*” indicates intermediate combinations. Distance  $L_z$  varied from  $L_z=120-L$  [cm] for cases with  $B=14$  cm, and  $L_z=185-L$  [cm] for cases with  $B=30$  cm.

Flow depths in both flumes were measured at various locations using point-gauge with 0.1 mm precision. Flow depth  $h_a$  was measured at the main channel centerline 25 cm upstream of each side weir to provide measurement in the upstream section where the water surface was parallel to the main channel bed. For tests in the narrower flume additional measurements with non-contact visualization technique were performed to determine longitudinal profiles of water surface along the side weir. A green laser (100 mW, 530–532 nm) was used to illuminate a thin vertical layer of the flow. With the flume obscured, the photos of different sections parallel to the main channel centerline were taken from perpendicularly positioned digital camera. Acquired photos were examined with the Matlab to determine grayscale values of pixels. Resulting water surface profile was calculated as a line of pixels separating a green and a black part of each photo.

Velocity fields were determined for tests in the narrower flume only, using recordings of electrolysis-generated hydrogen bubbles, as described by Novak et al. [14]. An attempt was made to use this technique in  $B=30$  cm flume as well, but it turned out a more powerful DC source would be needed to produce an adequate layer of hydrogen bubbles. The principles of both visualization techniques used for test runs in the narrow flume are compared in Fig. 3.

Calibration tests were performed as described in Novak et al. [14] to achieve good agreement between measured surface velocity components and observed floats. The uncertainty of average velocity measurements  $u_c^*(\bar{u})$  was  $u_c^*(\bar{u})=2\%$ .

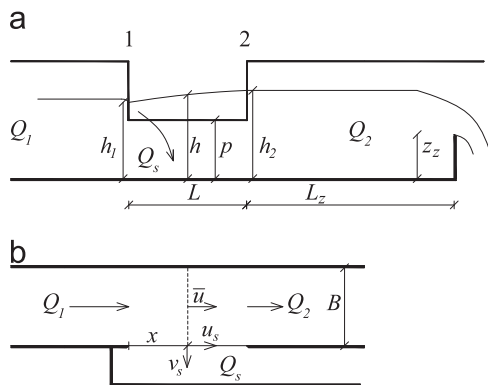


Fig. 1. Definition sketch of subcritical flow over rectangular side weir: (a) longitudinal section, (b) plan.

**Table 1**  
Main test parameters.

var.	<b>B=14, L=10, p=7.5 [cm]</b>			<b>B=14, L=15, p=7.5 [cm]</b>			<b>B=14, L=15, p=10 [cm]</b>		
	$z_c$ [cm]	$Q_t$ [l/s]	$Q_s$ [l/s]	$z_c$ [cm]	$Q_t$ [l/s]	$Q_s$ [l/s]	$z_c$ [cm]	$Q_t$ [l/s]	$Q_s$ [l/s]
var 0	5.29	3.97	0.84	5.28	4.52	1.24	6.91	5.45	1.09
Q-	5.29	3.31	0.84	5.28	3.82	1.09	6.90	4.54	0.89
Q-*	5.29	3.68	0.88	5.28	4.19	1.14	6.90	4.99	0.96
Q+*	5.30	4.38	0.85	5.28	4.94	1.32	6.90	5.73	1.22
Q+	5.30	4.71	0.92	5.28	5.30	1.34	6.90	6.06	1.29
Z-	4.22	3.97	0.46	4.21	4.50	0.63	6.00	5.42	0.78
Z-*	4.83	3.95	0.65	4.90	4.49	0.95	6.36	5.45	0.89
Z+*	5.71	3.95	1.02	6.20	4.50	1.65	7.37	5.45	1.32
Z+	6.12	3.96	1.22	6.57	4.50	1.87	8.10	5.46	1.65
var.	<b>B=14, L=20, p=10 [cm]</b>			<b>B=14, L=20, p=12 [cm]</b>			<b>B=14, L=25, p=12 [cm]</b>		
	$z_c$ [cm]	$Q_t$ [l/s]	$Q_s$ [l/s]	$z_c$ [cm]	$Q_t$ [l/s]	$Q_s$ [l/s]	$z_c$ [cm]	$Q_t$ [l/s]	$Q_s$ [l/s]
var 0	6.41	6.01	1.29	8.10	6.62	1.35	7.33	6.94	1.36
Q-	6.41	5.29	1.05	8.10	5.45	0.83	7.33	6.34	1.09
Q-*	6.41	5.69	1.14	8.10	6.14	1.10	7.33	6.59	1.21
Q+*	6.41	6.34	1.39	8.10	7.07	1.51	7.33	7.45	1.61
Q+	6.41	6.62	1.52	8.10	7.59	1.83	7.33	7.83	1.79
Z-	5.80	6.02	0.98	7.19	6.68	1.06	6.78	6.94	1.07
Z-*	6.11	6.02	1.14	7.78	6.64	1.19	7.70	6.91	1.52
Z+*	6.81	6.03	1.50	8.37	6.66	1.63	8.28	6.96	1.82
Z+	7.15	6.01	1.64	8.83	6.61	1.85	8.76	6.94	2.07
var.	<b>B=30, L=50, p=12 [cm]</b>			<b>B=30, L=75, p=14 [cm]</b>			<b>B=30, L=100, p=20 [cm]</b>		
	$z_c$ [cm]	$Q_t$ [l/s]	$Q_s$ [l/s]	$z_c$ [cm]	$Q_t$ [l/s]	$Q_s$ [l/s]	$z_c$ [cm]	$Q_t$ [l/s]	$Q_s$ [l/s]
1	6.95	20.00	6.04	8.15	27.90	10.01	12.80	34.60	14.82
2	10.21	19.85	9.99	10.87	27.90	14.30	13.63	34.60	16.23
3	13.57	18.75	13.49	12.73	27.90	16.96	15.79	34.55	19.38
4	8.49	17.10	6.67	10.02	25.00	11.49	13.73	30.20	13.43
5	10.14	17.20	8.68	12.85	25.20	15.89	14.21	30.20	14.57
6	11.58	17.10	10.19	15.19	25.18	19.01	15.12	30.20	15.29
7	8.70	14.25	5.46	10.12	22.00	9.95	16.81	25.30	15.41
8	10.08	14.25	7.03	13.00	22.20	14.14	17.85	25.30	16.66
9	11.68	14.05	8.45	14.88	22.10	16.35	19.10	25.20	18.39

**Table 2**  
Ranges of measured parameters.

Authors	B [cm]	L [cm]	p [cm]	S <sub>0</sub> [%]	Q <sub>t</sub> [l/s]	B/L [-]	Fr <sub>1</sub> [-]	Q <sub>s</sub> /Q <sub>t</sub> [-]
Novak et al. [14]	14	10–25	7.5–12	0.05	4–7.8	0.56–1.4	0.28–0.34	0.20–0.27
Present study	14, 30	10–100	7.5–20	0.05	3.31–34.6	0.3–1.4	0.23–0.41	0.12–0.76

### 3. Results and discussion

#### 3.1. Velocity ratio $u_s/\bar{u}$ and velocity distribution coefficients $\alpha$ and $\beta$

Our previous work (Novak et al. [14]) discussed  $u_s/\bar{u}$ ,  $\alpha$ , and  $\beta$  for variants labeled var0, Q-, and Q+ for all 6 side weirs in B=14 cm channel, amounting to 18 test (Table 1). In the present study, velocity fields were measured for additional 12 test runs, covering variants Z- and Z+ for all 6 side weirs in B=14 cm channel (Table 1). Results from these 12 runs were in accordance with conclusions presented by Novak et al. [14], and are given in Table 3 and Table 4.

#### 3.2. Velocity ratio $V_s/u_s$

Measured velocity ratio  $v_s/u_s$  can be used to define deviation angle of flow  $Q_s$ , usually given only in terms of average velocity of the approach flow and the velocity of flow  $Q_s$  over the brink (e.g. Emiroglu et al. [4]). Velocity components  $u_s$  and  $v_s$  of the flow  $Q_s$  were examined for all 30 tests discussed in previous section, to take the full advantage of the non-invasive visualization method.

Values of the ratio  $v_s/u_s$ , given as a function of location  $x/L$  along the side weir, increased for all test runs, ranging from values 0.1 to 0.6, as Table 3 and Table 4 show. This increasing trend was in accordance with well established observations of side overflow deviation angle by previous authors. In our experiments measured ratios  $v_s/u_s$  for various side weirs increased quite rapidly in the first section of the side weir, i.e. along  $x/L \leq 0.25$ , while  $v_s/u_s$  then changed less along the crest from  $x/L=0.25$  to  $x/L=1$ , as Fig. 4 shows.

#### 3.3. Energy grade line

In relation to the energy and momentum approach, mentioned in the introduction, the non-contactly measured water surface profiles  $h$ , average velocities  $\bar{u}$  and velocity distribution coefficients  $\alpha$  were used to determine energy grade line  $E=h+\alpha^2\bar{u}^2/2g$  in various cross sections. While water surface elevation increased along the side weir, line  $E$  remained parallel to the main channel bed for all considered test runs. This indicates that only friction losses were present, as proposed by the energy approach.

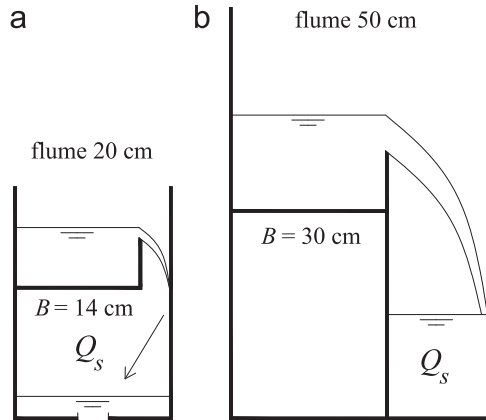


Fig. 2. Cross sections of side weirs for both sets of experiments: (a)  $Q_s$  flows into water tank, (b)  $Q_s$  is measured with a thin-plate weir in the lateral channel.

However, velocity fields were clearly non-uniform (values  $\alpha$  from 1.01 to 1.15), as assumed by the momentum approach.

### 3.4. Discharge coefficient $C_d$

Inserting measured values from all 81 tests into  $C_d$  equations by various authors gave values  $C_d$  in the range from  $C_d=0.34$  (using equation by Borghei et al. [7]) to  $C_d=0.72$  (using equation by Singh et al. [5]). An even wider range of  $C_d$  from equations by various authors was reported by Emiroglu et al. [3] for  $L/B=3$  case, which is similar to our  $L = 100$  cm,  $B=30$  cm case. For each observed variant of our experiments, values  $C_d$  were obtained from Eq. (1) and denoted as  $C_{d,mer}$  to indicate that they were calculated from measured overflows  $Q_s$  and flow depths  $h$  along the observed reach. Value  $h$  in Eq. (1) was taken as an average flow depth  $h_{avg}$  along the side weir, measured in the main channel centerline. Values  $C_{d,mer}$  are shown in Table 5.

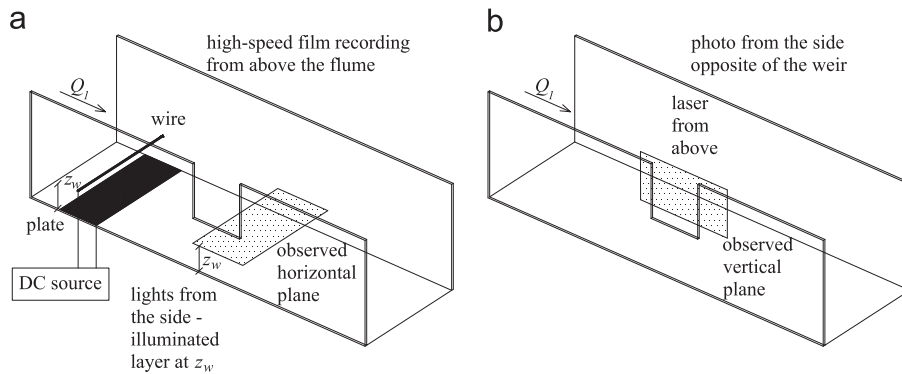


Fig. 3. Sketch of experimental set-up: (a) measurements of velocity fields using electrolysis-generated hydrogen bubbles, (b) determination of water surface longitudinal profiles using photos of laser-induced sections.

Table 3  
Main results regarding velocity fields—part 1.

var	$B=14, L=10, p=7.5$ [cm]						$B=14, L=15, p=7.5$ [cm]						$B=14, L=15, p=10$ [cm]					
	$x/L$	$h/p$	$\bar{u}_s/\bar{u}$	$\alpha$	$\beta$	$v_s/u_s$	$x/L$	$h/p$	$\bar{u}_s/\bar{u}$	$\alpha$	$\beta$	$v_s/u_s$	$x/L$	$h/p$	$\bar{u}_s/\bar{u}$	$\alpha$	$\beta$	$v_s/u_s$
var0	0	1.33	1.03	1.02	1.01	0.32	0	1.33	0.98	1.01	1.00	0.27	0	1.26	1.08	1.02	1.01	0.16
	0.25	1.33	1.09	1.03	1.01	0.45	0.25	1.34	1.00	1.02	1.01	0.34	0.25	1.26	1.02	1.01	1.00	0.34
	0.5	1.33	1.04	1.03	1.01	0.48	0.5	1.35	1.03	1.02	1.01	0.26	0.5	1.27	1.08	1.04	1.01	0.51
	0.75	1.34	1.07	1.04	1.01	0.52	0.75	1.36	1.00	1.05	1.02	0.24	0.75	1.27	1.07	1.03	1.01	0.34
	1	1.34	1.10	1.05	1.02	0.57	1	1.37	1.02	1.04	1.01	0.35	1	1.28	1.06	1.03	1.01	0.41
Q+	0	1.38	0.98	1.02	1.01	0.30	0	1.39	1.09	1.05	1.02	0.24	0	1.28	0.94	1.01	1.00	0.39
	0.25	1.39	1.05	1.03	1.01	0.30	0.25	1.40	1.05	1.05	1.02	0.39	0.25	1.29	0.98	1.02	1.01	0.39
	0.5	1.39	1.04	1.04	1.01	0.40	0.5	1.41	1.13	1.07	1.02	0.38	0.5	1.29	1.01	1.02	1.01	0.39
	0.75	1.39	1.08	1.06	1.02	0.40	0.75	1.42	1.09	1.09	1.03	0.44	0.75	1.30	1.07	1.02	1.01	0.25
	1	1.39	1.11	1.06	1.02	0.42	1	1.43	1.20	1.13	1.04	0.40	1	1.31	1.11	1.02	1.01	0.28
Q-	0	1.27	1.00	1.03	1.01	0.31	0	1.29	1.11	1.04	1.01	0.26	0	1.21	1.00	1.01	1.00	0.32
	0.25	1.27	1.00	1.02	1.01	0.36	0.25	1.29	1.15	1.05	1.02	0.29	0.25	1.21	1.04	1.01	1.00	0.35
	0.5	1.27	0.98	1.02	1.01	0.37	0.5	1.30	1.14	1.05	1.02	0.30	0.5	1.22	1.00	1.01	1.00	0.35
	0.75	1.28	0.98	1.02	1.01	0.38	0.75	1.31	1.05	1.06	1.02	0.38	0.75	1.22	1.05	1.03	1.01	0.29
	1	1.28	1.03	1.04	1.01	0.34	1	1.31	1.03	1.05	1.02	0.37	1	1.22	0.99	1.03	1.01	0.34
Z+	0	1.40	1.18	1.08	1.03	0.34	0	1.44	1.11	1.04	1.01	0.28	0	1.33	1.06	1.03	1.01	0.35
	0.25	1.40	1.21	1.06	1.02	0.47	0.25	1.44	1.12	1.05	1.02	0.35	0.25	1.34	1.12	1.03	1.01	0.39
	0.5	1.41	1.13	1.05	1.02	0.55	0.5	1.45	1.06	1.04	1.01	0.47	0.5	1.34	1.09	1.03	1.01	0.57
	0.75	1.41	1.14	1.06	1.02	0.59	0.75	1.46	1.10	1.06	1.02	0.44	0.75	1.35	1.02	1.03	1.01	0.45
	1	1.41	1.16	1.06	1.02	0.59	1	1.47	1.07	1.04	1.01	0.34	1	1.36	1.02	1.03	1.01	0.44
Z-	0	1.22	0.94	1.02	1.01	0.18	0	1.24	1.01	1.05	1.02	0.14	0	1.20	1.05	1.04	1.01	0.27
	0.25	1.22	0.99	1.01	1.00	0.25	0.25	1.24	0.99	1.05	1.02	0.18	0.25	1.20	1.12	1.06	1.02	0.29
	0.5	1.22	0.98	1.02	1.01	0.33	0.5	1.25	1.07	1.08	1.03	0.30	0.5	1.20	1.17	1.10	1.04	0.25
	0.75	1.22	1.02	1.04	1.01	0.38	0.75	1.26	1.04	1.08	1.03	0.25	0.75	1.20	1.17	1.12	1.04	0.22
	1	1.22	1.06	1.04	1.01	0.31	1	1.26	1.09	1.08	1.03	0.25	1	1.21	1.22	1.16	1.05	0.22

**Table 4**  
Main results regarding velocity fields—part 2.

var	<b>B=14, L=20, p=10 [cm]</b>						<b>B=14, L=20, p=12 [cm]</b>						<b>B=14, L=25, p=12 [cm]</b>					
	x/L	h/p	$\bar{u}_s/\bar{u}$	$\alpha$	$\beta$	$v_s/u_s$	x/L	h/p	$\bar{u}_s/\bar{u}$	$\alpha$	$\beta$	$v_s/u_s$	x/L	h/p	$\bar{u}_s/\bar{u}$	$\alpha$	$\beta$	$v_s/u_s$
var0	0	1.24	1.05	1.03	1.01	0.13	0	1.19	1.00	1.03	1.01	0.31	0	1.16	0.97	1.04	1.02	0.29
	0.25	1.24	1.14	1.08	1.03	0.39	0.25	1.19	1.15	1.13	1.04	0.45	0.25	1.17	1.10	1.08	1.03	0.32
	0.5	1.25	1.06	1.02	1.01	0.33	0.5	1.20	1.04	1.03	1.01	0.42	0.5	1.17	1.04	1.04	1.01	0.29
	0.75	1.25	1.08	1.02	1.01	0.33	0.75	1.20	1.10	1.08	1.03	0.47	0.75	1.18	1.17	1.17	1.06	0.33
	1	1.26	1.19	1.05	1.02	0.37	1	1.20	1.04	1.05	1.02	0.49	1	1.18	1.14	1.09	1.03	0.35
Q+	0	1.26	1.01	1.04	1.01	0.31	0	1.22	1.13	1.07	1.02	0.30	0	1.19	1.10	1.06	1.02	0.23
	0.25	1.27	1.09	1.07	1.02	0.37	0.25	1.23	1.20	1.09	1.03	0.41	0.25	1.20	1.13	1.08	1.03	0.36
	0.5	1.27	1.06	1.02	1.01	0.38	0.5	1.23	1.15	1.07	1.02	0.42	0.5	1.21	1.11	1.04	1.01	0.29
	0.75	1.28	1.11	1.05	1.02	0.38	0.75	1.24	1.20	1.09	1.03	0.45	0.75	1.21	1.17	1.07	1.03	0.33
	1	1.29	1.13	1.04	1.01	0.44	1	1.24	1.24	1.08	1.03	0.50	1	1.22	1.15	1.06	1.02	0.36
Q-	0	1.20	1.07	1.04	1.01	0.15	0	1.14	1.03	1.03	1.01	0.37	0	1.14	1.11	1.05	1.02	0.18
	0.25	1.20	1.08	1.03	1.01	0.35	0.25	1.15	1.12	1.04	1.01	0.41	0.25	1.15	1.14	1.06	1.02	0.21
	0.5	1.21	1.06	1.02	1.01	0.30	0.5	1.15	1.08	1.02	1.01	0.35	0.5	1.16	1.10	1.04	1.01	0.24
	0.75	1.21	1.11	1.05	1.02	0.36	0.75	1.15	1.09	1.04	1.01	0.39	0.75	1.16	1.21	1.06	1.02	0.28
	1	1.22	1.13	1.05	1.02	0.41	1	1.16	1.11	1.04	1.01	0.35	1	1.16	1.25	1.05	1.02	0.22
Z+	0	1.27	1.06	1.03	1.01	0.23	0	1.24	1.12	1.06	1.02	0.22	0	1.22	1.15	1.11	1.04	0.15
	0.25	1.28	1.07	1.06	1.02	0.42	0.25	1.24	1.21	1.07	1.02	0.43	0.25	1.23	1.18	1.12	1.04	0.37
	0.5	1.29	1.03	1.03	1.01	0.43	0.5	1.25	1.05	1.04	1.01	0.47	0.5	1.23	1.12	1.06	1.02	0.32
	0.75	1.30	1.07	1.04	1.02	0.37	0.75	1.26	1.09	1.04	1.02	0.53	0.75	1.24	1.21	1.12	1.04	0.40
	1	1.31	1.07	1.02	1.01	0.40	1	1.27	1.09	1.06	1.02	0.49	1	1.24	1.24	1.08	1.03	0.43
Z-	0	1.20	1.07	1.02	1.01	0.18	0	1.16	1.03	1.04	1.01	0.30	0	1.14	1.07	1.08	1.03	0.17
	0.25	1.20	1.11	1.05	1.02	0.29	0.25	1.17	1.11	1.06	1.02	0.32	0.25	1.15	1.13	1.05	1.02	0.23
	0.5	1.21	1.06	1.02	1.01	0.29	0.5	1.18	1.08	1.03	1.01	0.30	0.5	1.15	1.15	1.06	1.02	0.21
	0.75	1.22	1.17	1.08	1.03	0.34	0.75	1.18	1.19	1.06	1.02	0.32	0.75	1.15	1.22	1.12	1.04	0.20
	1	1.23	1.30	1.10	1.03	0.36	1	1.19	1.24	1.03	1.01	0.37	1	1.16	1.29	1.08	1.03	0.27

In Table 5, notation of test runs in the wider flume gives information about inflow in l/s and spilling ratio  $Q_s/Q_T$ , with notation “a” representing the smallest ratio (around  $Q_s/Q_T=0.3$  to 0.4) and notation “c” indicating the greatest ratio (around  $Q_s/Q_T=0.6$  to 0.7). Exceedingly high values of  $C_{d,mer}$  and high standard deviation values (STDEV) for the smallest two side weirs (L10\_p7.5 and L15\_p7.5 in Table 5) suggested that some flow measurements for these two configurations were probably not reliable enough. For this reason, results from L10\_p7.5 and L15\_p7.5 side weirs were not used for the formulation of  $C_d$  equation. Consequently, the range of validity of  $C_d$  equation was adjusted. With the elimination of results from the smallest two observed side weirs, 63 results remained, and the experimental limits were the following:  $B=14$  to 30 cm,  $L=15$  to 100 cm,  $p=10$  to 20 cm,  $Q_T=4.54$  to 34.6 l/s,  $Q_s=0.89$  to 19.38 l/s, and  $Fr_1=0.23$  to 0.41.

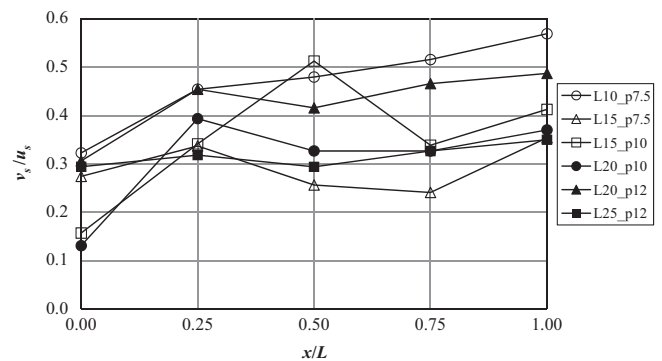
Dimensional analysis was used to formulate an equation for  $C_d$  on the basis of our experimental data. In doing so, various dimensionless ratios were considered. Finally, the following power-law term was found to be the most suitable:

$$C_{d,mer} = K Fr_1^\delta (h_2/p)^\epsilon (B/L)^\zeta \quad (2)$$

with unknown constant  $K$ , and unknown exponents  $\delta$ ,  $\epsilon$ , and  $\zeta$ . Eq. (2) formed a system of equations when the measured quantities were inserted. A logarithm (ln) operation was used to obtain a system of linear equations for unknown variables. This system was solved in Matlab to determine the mentioned unknowns and resulting values  $C_d$ . From all the data from test runs L15\_p10 to L100\_p20, the following equation was obtained:

$$C_d = 0.4689 Fr_1^{-0.166} (h_2/p)^{-0.047} (B/L)^{-0.135} \quad (3)$$

The correlation between  $C_{d,mer}$  and  $C_d$  from Eq. (3) was observed in terms of correlation coefficient  $r$ . To achieve  $r=0.85$



**Fig. 4.** Values  $v_s/u_s$  for observed side weirs in narrow flume ( $B=14$  cm)—variants labeled var0 only.

value, only additional 2 out of 63 considered experimental results had to be eliminated.

The Eq. (3) is suitable for practical applications, as it includes  $Fr_1$  (in accordance with the great majority of equations by other authors), ratio  $h_2/p$  (e.g. like May et al. [13]), and  $B/L$  (e.g. like Borghei et al. [7], and Emiroglu et al. [4]). For the investigated side weirs L15\_p10 to L100\_p20, the Eq. (3) gives  $C_d$  values which are similar to ones calculated from the following equations:

$$C_d = 0.650 - 0.149 [(h_2 - p)/p]^{0.0868} [L/(h_2 - p)]^{-0.303} (h_2/p)^{0.149}$$

by May et al. [13] and

$$C_d = 0.81 - 0.6 Fr_1$$

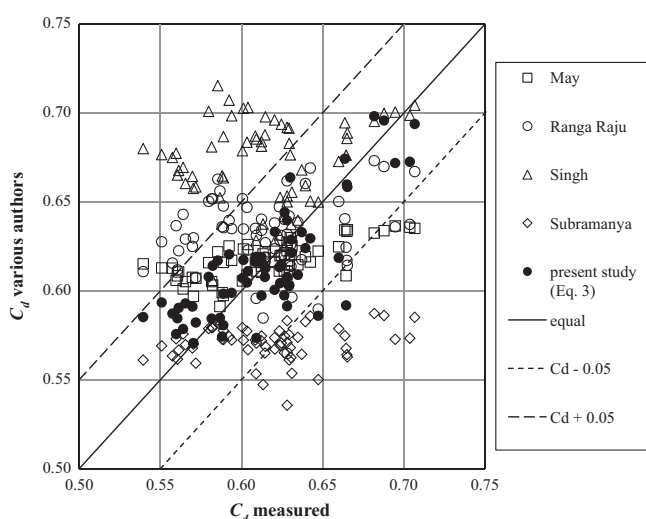
by Ranga Raju et al. [18].

When our experimental data was inserted in the following equations:

$$C_d = 0.33 - 0.18 Fr_1 + 0.49 (p/h_1)$$

**Table 5**  
Values  $C_{d,mer}$  for all measured cases.

variant	Narrower flume ( $B=14$ cm)						Wider flume ( $B=30$ cm)					
	L10_p7.5	L15_p7.5	L15_p10	L20_p10	L20_p12	L25_p12	var.	L50_p12	var.	L75_p14	var.	L100_p20
var0	0.72	0.66	0.56	0.56	0.63	0.61	20_a	0.65	28_a	0.63	35_a	0.67
Q-	0.97	0.73	0.63	0.59	0.59	0.59	20_b	0.63	28_b	0.61	35_b	0.66
Q-*	0.86	0.67	0.58	0.55	0.58	0.60	20_c	0.58	28_c	0.61	35_c	0.63
Q+*	0.63	0.65	0.59	0.56	0.61	0.61	17_a	0.62	25_a	0.63	30_a	0.69
Q+	0.63	0.56	0.57	0.57	0.66	0.63	17_b	0.63	25_b	0.63	30_b	0.70
Z-	0.74	0.56	0.61	0.54	0.59	0.60	17_c	0.62	25_c	0.62	30_c	0.66
Z-*	0.72	0.60	0.56	0.56	0.62	0.62	14_a	0.66	22_a	0.64	25_a	0.71
Z+*	0.75	0.65	0.59	0.57	0.60	0.61	14_b	0.64	22_b	0.63	25_b	0.69
Z+	0.77	0.68	0.59	0.57	0.60	0.61	14_c	0.64	22_c	0.63	25_c	0.68
AVG	0.76	0.64	0.59	0.56	0.61	0.61		0.63		0.63		0.68
STDEV	0.107	0.057	0.022	0.014	0.026	0.011		0.022		0.009		0.024



**Fig. 5.**  $C_d$  measured (i.e. calculated from Eq. (1)) versus  $C_d$  by various authors, for 63 experimental data from L15\_p10 to L100\_p20 side weirs.

by Singh et al. [5] and

$$C_d = 0,864 [(1 - Fr_1^2) / (2 + Fr_1^2)]^{0.5}$$

by Subramanya and Awasthy [19], the considerable number of calculated  $C_d$  values fell out of the  $\pm 0.05$  region (dashed lines in Fig. 5). For the same experimental data the equations by authors, other than those shown in Fig. 5, gave values  $C_d$  that deviated considerably from  $C_d$  of Eq. (3).

#### 4. Conclusions

Our previous work concerning subcritical flow at sharp-crested rectangular side weirs in rectangular straight channels was extended to cover a wider range of measured geometrical and hydraulic parameters. Both longitudinal water surface profiles and velocity fields at horizontal planes were determined using non-invasive visualization techniques, i.e. photos of laser-induced sections and films of electrolysis-generated hydrogen bubbles.

The present study confirmed velocity fields at side weir were clearly non-uniform with velocity ratios  $u_s/\bar{u}$  and  $v_s/u_s$  increasing along the side weir from 1 to 1.2 and 0.1 to 0.6, respectively, and kinetic energy correction coefficient  $\alpha$  ranging from 1.01 to 1.15 along the side weir. However, the energy grade line, calculated

from the measured flow depths and velocities, remained parallel to the main channel bed for all considered test runs.

New phenomenological equation for the discharge coefficient  $C_d$  of the rectangular sharp-crested side weir in narrow flume was developed using dimensional analysis from experimental data covering the following range of parameters:  $B=14$  and 30 cm,  $L=15$  to 100 cm,  $p=10$  to 20 cm,  $Q_I=4.54$  to 34.6 l/s,  $Q_S=0.89$  to 19.38 l/s and  $Fr_I=0.23$  to 0.41. The proposed equation gives  $C_d$  values which are in good agreement with two other studies in literature.

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